

SOLAR NEUTRINOS

(with a tribute to John N. Bahcall)

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ABSTRACT

John N. Bahcall championed solar neutrino physics for many years. Thanks to his pioneering and long-lasting contributions, this field of research has not only reached maturity, but has also opened a new window on physics beyond the standard electroweak model through the phenomenon of neutrino flavor oscillations. We briefly outline some recent accomplishments in the field, and also discuss a couple of issues that do not seem to fit in the “standard picture,” namely, the chemical controversy at the solar surface, and possible implications of recent gallium radioactive source experiments.



Figure 1: John N. Bahcall explains “Why the Sun shines” in his *Lectio Magistralis*, before receiving the *Laurea Honoris Causa* in Physics at the University of Milan, Italy (May 6th, 2004).

1. John N. Bahcall (1934–2005): Memories of our interactions

John Bahcall championed solar neutrino physics for many years. His countless seminars on the solar neutrino problem as a window to new physics are probably responsible for the interest in neutrino physics of many participants to this Workshop (NO-VE 2006) — and they were definitely so for myself, in the early 1990’s. [Milla Baldo Ceolin is also responsible in part... having invited me to give my very first talk on solar neutrinos in the 1993 Workshop on Neutrino Telescopes in Venice.] At that time I was gradually shifting my main interests from electroweak precision physics to neutrino physics, and John’s classic book on “Neutrino Astrophysics” ¹⁾ was a major guidance in this new (for me) field of research.

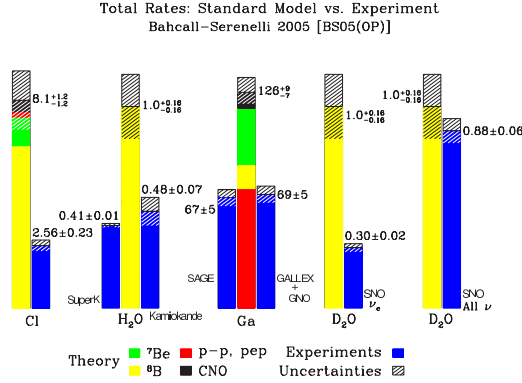


Figure 2: Solar neutrino event rates: SSM expectations vs data (2005). From John’s website ⁶⁾.

More direct interactions between John and me started in 1994, first with scientific correspondence on his famous “1,000 Standard Solar Models (SSM)” ²⁾ (which were used to deal with neutrino flux errors and correlations ³⁾), and then with his acceptance of my former PhD student Eligio Lisi as INFN postdoc in his group at the Institute of Advanced Studies in Princeton.

Interactions with John have always been both scientifically interesting and personally enjoyable. In particular, I remember with great pleasure his participation to our Neutrino Oscillation Workshop 2000 (Otranto, Italy) ⁴⁾, where he managed to come — and to present preliminary new results from the so-called “BP 2000” (Bahcall-Pinsonneault) SSM ⁵⁾ — despite his many other obligations in that period.

With the same great pleasure I remember his “Laurea Honoris Causa” at the University of Milan, Italy, in May 2004 (see John’s picture in Fig. 1), where several participants to this workshop, including myself, were very happy to be in the Laurea committee celebrating John’s outstanding career. That event, as many others, witnessed the close bond of friendship between John and all of us in Italy.

These are just a few facets, from a personal perspective, of his long-life commitment to science, and to the scientific community. Any of us could add countless examples of such commitment. He was really a leading scientist in our field, and we all miss him greatly. But, his greatest accomplishment remains with us: solar neutrinos as a window to new physics.

2. Solar neutrino physics: Established facts

Four decades (1965-2005) of enduring theoretical efforts, difficult experiments, and extraordinary achievements in solar ν researches are well summarized by one of John’s favorite viewgraphs ⁶⁾, reported in Fig. 2. The graph shows the comparison between SSM expectations (highest bars) and data (blue bars) for the observation of the solar neutrino flux in the Chlorine (Cl) ⁷⁾, Gallium (Ga) ^{8,9,10)}, Super-Kamiokande (H₂O) ^{11,12)} and SNO (D₂O) ^{13,14,15)}, together with their error bars (shaded areas) and

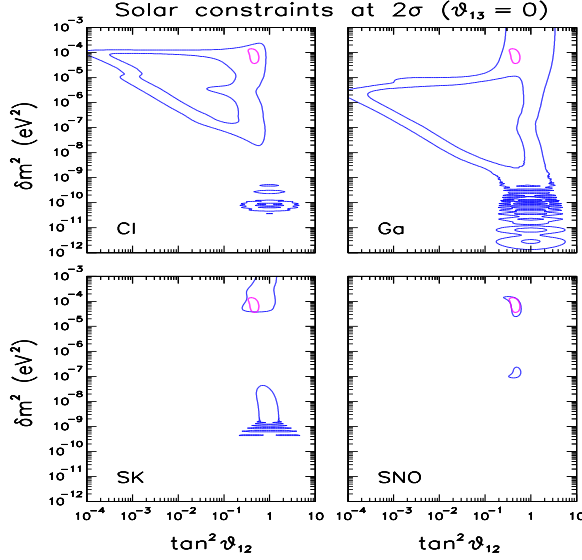


Figure 3: Separate (thin) and combined (thick) bounds on the solar neutrino oscillation parameters from Ga, Cl, SK, and SNO data at 2σ (95% C.L. for 1 d.o.f.), for the case $\theta_{13} = 0$ ¹⁹⁾.

neutrino flux components (in different colors). All but the rightmost bars show a deficit of measured solar ν_e events as compared to no-oscillation expectations (the famous solar neutrino problem); the rightmost bars shows instead no deficit in the “all ν ” flux from the Sun ($\nu_e + \nu_\mu + \nu_\tau$), implying that the ν_e deficit *must* be due to $\nu_e \rightarrow \nu_{\mu,\tau}$ flavor transitions ^{13,14)}. This phenomenon, theorized long ago by Bruno Pontecorvo ¹⁶⁾, requires massive and mixed neutrinos, and thus appears to be our first window open beyond the minimal standard model of the electroweak interactions. The current solar ν constraints on this phenomenon are briefly summarized below.

2.1. Status of solar neutrino oscillation parameters

The dominant flavor transition (“oscillation”) parameters for solar neutrinos are, in standard notation ¹⁷⁾, the squared mass difference $\delta m^2 = m_2^2 - m_1^2$ and the mixing angle θ_{12} . Subdominant effects can be induced by the mixing angle θ_{13} , which is known to be small ¹⁸⁾. Assuming $\theta_{13} = 0$, the current solar neutrino constraints on the dominant parameters are shown in Fig. 3 ^{19,20)}, both from separate experiments (four panels) and in combination (thicker “potato” in each panel). The combination, universally known as “large mixing angle” (LMA) solution is currently: (1) unique (no multiple solutions); (2) highly consistent with each data set; and (3) dominated by the SNO experiment, and to a lesser extent by the SK experiment (which cuts the spurious low- δm^2 solution still allowed by SNO). Lower-energy data (Ga and Cl) play instead a role in constraining θ_{13} (see later).

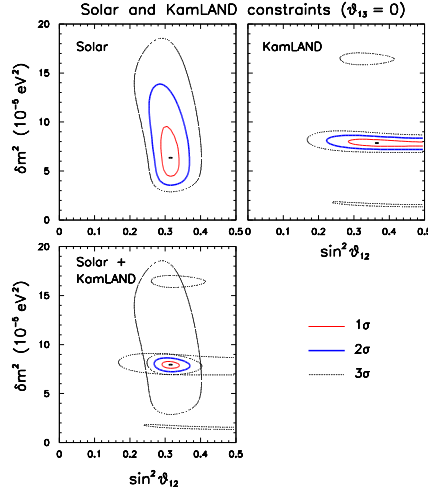


Figure 4: Contours of the regions allowed at 1, 2, and 3 σ (for 1 d.o.f.) by solar ν data (upper left panel), KamLAND data (upper right panel) and their combination (lower panel) in the case $\theta_{13} = 0$.

2.2. Consistency with Standard Solar Model predictions

As already remarked, the constraints on the leading parameters $(\delta m^2, \theta_{12})$ are dominated by SNO and SK data, sensitive to the high-energy (^8B) component of the solar neutrino flux. One can perform an analysis of SNO and SK data independent of both the SSM and the details of the oscillation phenomenon²¹⁾, showing that such data constrain the total ^8B neutrino flux in a range consistent with SSM predictions¹⁹⁾ within 1 σ . Moreover, the experimental error on this flux is a factor of ~ 2 smaller than the theoretical SSM one²²⁾, implying that the latter does not play anymore a crucial role in the determination of the LMA parameters. See also¹⁵⁾.

2.3. Consistency with KamLAND reactor neutrino data

The KamLAND observation of reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$ disappearance²³⁾ and associated spectral distortions²⁴⁾, driven by the same leading parameters $(\delta m^2, \theta_{12})$ as for solar ν_e , has strongly increased our confidence in ν oscillations. Figure 4¹⁹⁾ shows the very good consistency between solar and KamLAND constraints on the oscillation parameters, as well as their complementarity: solar ν data mainly fix $\sin^2 \theta_{12}$ (which is basically measured by SNO through the charged-to-neutral current event ratio CC/NC¹⁵⁾), while KamLAND data mainly fix δm^2 (through the spectral distortion pattern²⁴⁾). Their combination in Fig. 4 can be summarized (with $\pm 2\sigma$ errors) as:

$$\delta m^2 = 7.92(1 \pm 0.09) \times 10^{-5} \text{ eV}^2, \quad (1)$$

$$\sin^2 \theta_{12} = 0.314(1^{+0.18}_{-0.15}). \quad (2)$$

These $\pm 2\sigma$ ranges are not altered for $\theta_{13} \neq 0$ ¹⁹⁾ (not shown).

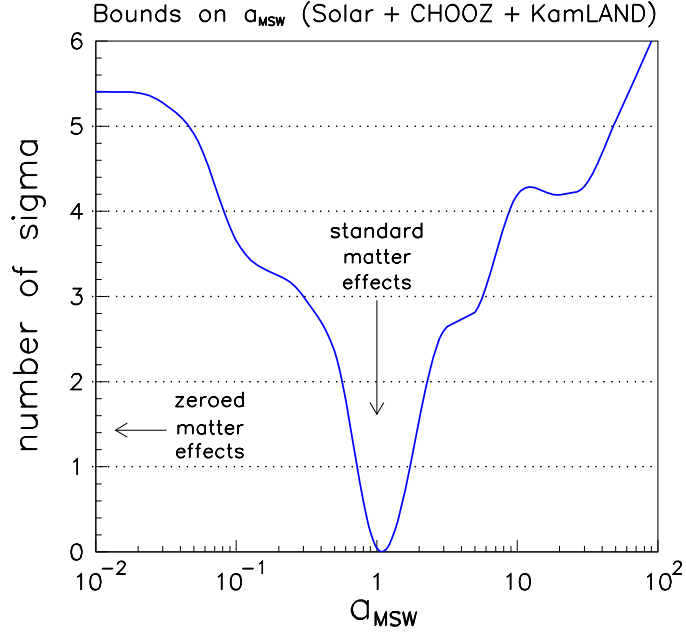


Figure 5: Evidence for MSW effects in matter from solar+reactor data ¹⁹⁾. See the text for details.

2.4. Consistency with expected matter effects

After the seminal works by Wolfenstein and by Mikheyev and Smirnov ²⁵⁾ (MSW), it has been realized that even if neutrinos are negligibly absorbed by matter (a G_F^2 effect), their oscillation phases can be significantly modified by background fermions (a G_F effect). The relevant function governing the MSW effect in ordinary matter is the so-called neutrino potential $V(x)$,

$$V(x) = \sqrt{2}G_F N_e(x) , \quad (3)$$

where $N_e(x)$ is the electron density at the position x along the ν trajectory. This potential must be definitely taken into account in the theoretical interpretation of solar ν data within the LMA solution (see ^{26,27)} for recent reviews).

One possible way to test the occurrence of the MSW effect is to allow a free normalization factor a_{MSW} for the potential,

$$V(x) \rightarrow a_{\text{MSW}} V(x) , \quad (4)$$

and let the data decide whether $a_{\text{MSW}} = 0$ (no effect) or $a_{\text{MSW}} = 1$ (standard effect).

Figure 5 shows the “number of sigmas” of a fit to all solar+reactor data, as a function of a_{MSW} ¹⁹⁾. The case of no effect is excluded at $> 5\sigma$, while the standard MSW effect is strongly favored, with amplitude constrained within a factor of ~ 2 at $\pm 2\sigma$. In a sense, this is an unconventional measurement of G_F (although not particularly accurate) through the phenomenon of neutrino oscillations in matter.

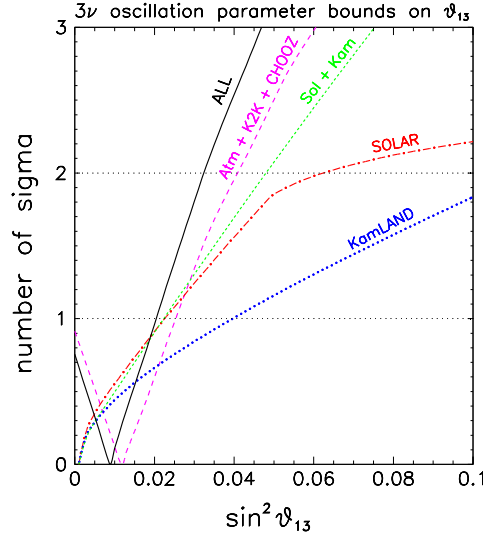


Figure 6: Global analysis within the 3ν mixing framework. Bounds on $\sin^2 \theta_{13}$ from various data sets with increasing constraining power: KamLAND, solar, KamLAND+solar, atmospheric+accelerator+CHOOZ, and all data combined. The overall preference for a nonzero value of θ_{13} is not statistically significant ($< 1\sigma$). From ^{19).}

2.5. Synthesis within the three-neutrino mixing framework

The previous results, although obtained for $\theta_{13} = 0$ (effective 2ν mixing), are not significantly altered for $\theta_{13} \neq 0$ (3ν mixing), due to the nontrivial consistency of all current ν oscillation data (both separately and in combination) for *small* values of θ_{13} . Indeed, although the current upper bounds on θ_{13} are dominated by the null results of the CHOOZ reactor experiment (in combination with atmospheric and accelerator data, see e.g., ¹⁹⁾), the solar neutrino constraints are only a factor ~ 2 weaker. From Fig. 6 one can derive, for instance, the following 2σ (95% C.L.) bounds on $\sin^2 \theta_{13}$:

$$\sin^2 \theta_{13} < 0.062 \text{ (solar) ,} \quad (5)$$

$$< 0.048 \text{ (solar + KamLAND) ,} \quad (6)$$

$$< 0.032 \text{ (all data) .} \quad (7)$$

Interestingly, the upper bounds on θ_{13} from “solar data only” are due to the interplay between “low-energy” (LE) data and “high-energy” (HE) data—say, Gallium vs SNO—which measure the solar ν_e survival probability in different limits ²⁸⁾:

$$P_{ee}^{\text{LE}} \simeq (1 - 2 \sin^2 \theta_{13})(1 - 0.5 \sin^2 2\theta_{12}) , \quad (8)$$

$$P_{ee}^{\text{HE}} \simeq (1 - 2 \sin^2 \theta_{13}) \sin^2 \theta_{12} . \quad (9)$$

From the above equations it follows that an increase of θ_{13} can be compensated by a decrease (increase) of θ_{12} in low-energy (high-energy) solar neutrino data; then, as θ_{13} grows, these diverging shifts in θ_{12} will eventually become inconsistent with each other and with the data, leading to the upper bound on θ_{13} in Eq. (5).

3. Solar neutrino physics: Are there little “cracks”?

The previous beautiful and solid facts should not make us overly confident in our current understanding of solar neutrino physics. Words of caution came from John Bahcall himself. At the Neutrino 2002 Conference, when the SSM predictions for the ^8B neutrino flux appeared to be convincingly confirmed by SNO, he stated that

“This is the first time in 40 years of giving talks in solar neutrinos that it seems to me that the people in the audience are more confident of the solar model predictions than I am.”

In addition, one of his favorite quotes (of much more general applicability...) was:

“Half of all three sigma results are wrong.”

These admonitions suggest an open-minded attitude: one should not be blind to small “cracks” that might open up in the beautiful solar neutrino construction.

3.1. Chemical controversy at the solar surface

One such “crack” (the metallicity problem) had a central role in John’s late interests. In a nutshell, it turns out that, when the newest metal abundances ²⁹⁾ are adopted as SSM input, the fractional difference between the sound speed profile predicted by the SSM and the one inferred from helioseismology becomes alarmingly large—while it used to be very small with “older” metallicity inputs ³⁰⁾. No clear solution is emerging for this metallicity problem, as also recognized in John’s last research paper ²²⁾. Indeed, while previous SSM papers by John always contained a “recommended” set of neutrino fluxes and errors, this one ²²⁾ leaves an open choice between (at least) two extreme cases for central values and errors.

Luckily, even in the worst case for helioseismology (“new” metal abundances with optimistically small errors), the estimated ^8B flux uncertainty in the corresponding SSM ²²⁾ is still a factor ~ 2 larger than the experimental one from SNO. Therefore (see the comments in Sec. 2.2) the solar neutrino bounds on the LMA parameters ($\delta m^2, \sin^2 \theta_{12}$) remain largely insensitive to the metallicity problem, whose disturbing effects seem to be confined to helioseismology so far. However, this problem might surface again in solar neutrino physics, should the theoretical SSM uncertainties become less conservative in the future.

3.2. Issues in gallium radioactive source experiments

Another small “discrepancy,” whose implications on solar neutrino parameters, to our knowledge, have not been discussed before, stems from a recent paper ³¹⁾ about the radioactive source experiment in SAGE, GALLEX and GNO. These experiments

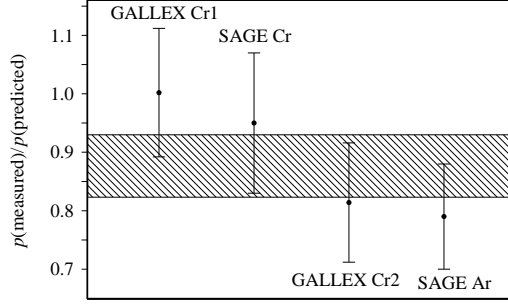


Figure 7: Ratio of measured to predicted event rates from radioactive source experiments in Ga detectors. The shaded band correspond to the combined result 0.88 ± 0.05 . From ³¹⁾.

measure the ν event rate in Gallium induced by radioactive ν_e sources with known intensity, and can thus test the theoretical cross section for ν_e absorption in Ga calculated by John Bahcall in ³²⁾. More precisely, only the low-energy range of the cross section (say, < 2 MeV) is tested, which is relevant for the so-called pp, pep, Be, N, and O contributions to the solar neutrino flux. The higher energy range (> 2 MeV), relevant for the B and hep contributions, is instead decoupled from the lower energy one ³³⁾ and is untested by the radioactive source experiments reported in ³¹⁾.

Figure 7 shows the various results reported in ³¹⁾, which can be combined as

$$\text{Ga rate (radioactive source)} : \frac{\text{measured}}{\text{predicted}} = 0.88 \pm 0.05 \quad (10)$$

(shaded band in the figure), where the total error is at 1σ . In other words, according to the claim in ³¹⁾, the theoretical Ga cross-section in ³²⁾ might be overestimated by a factor $1/(0.88 \pm 0.05)$ (at least at low energy): not a negligible effect ($> 2\sigma$).

It is then tempting to see what happens if the estimated Ga cross section in ³²⁾ is “renormalized” ad hoc by a factor 0.88 ± 0.05 for energies below ~ 2 MeV. We can expect, from the comments to Eqs. (8) and (9), that a significant change in the Ga predictions will at least alter the bounds on θ_{13} derived from solar neutrino data [Eq. (5)]. In the following we revisit such bounds, by comparing the two cases with “standard Ga cross section” and with ad hoc “renormalized Ga cross section” [$\sigma_{\text{Ga}} \rightarrow \sigma_{\text{Ga}} \times (0.88 \pm 0.05)$ for $E_\nu < 2$ MeV]. Our results should be intended as a preliminary exploration of this issue.

Figure 8 shows two key quantities as a function of $\sin^2 \theta_{12}$: the Ga event rate in solar ν experiments, normalized to SSM ^{34,19)} expectations, and the CC/NC event ratio in SNO. These quantities, representative of low- and high-energy ν observables, are plotted both for $\sin^2 \theta_{13} = 0$ (solid) and $\sin^2 \theta_{13} = 0.05$ (dashed). The horizontal bands represent the corresponding experimental data at $\pm 1\sigma$. The two panels differ only for the Ga cross section: standard (left) and “renormalized” at low energy (right); therefore, only the Ga observables change from left to right (not CC/NC).

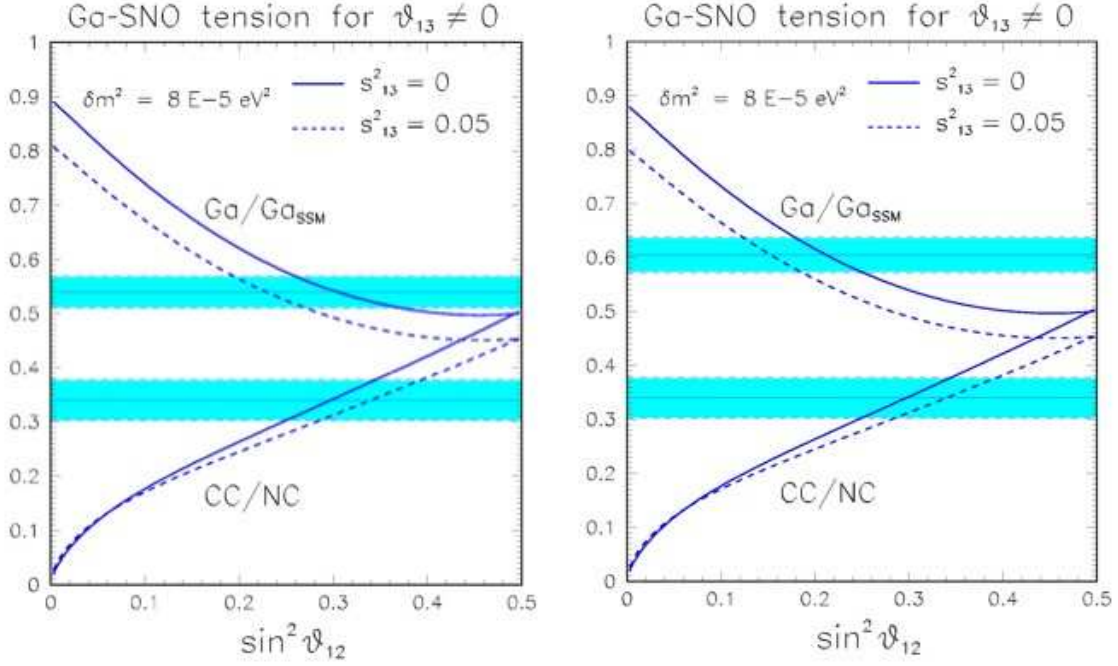


Figure 8: Comparison of data and prediction for Gallium and SNO observables, for standard and “renormalized” Ga cross section (left and right plots, respectively). See the text for details.

In the left plot of Fig. 8 one can see that, for $\sin^2 \theta_{12} \simeq 0.3$, both solid curves agree very well with the data, i.e., there is high consistency between Ga and SNO information at $\sin^2 \theta_{13} = 0$. For $\sin^2 \theta_{13} = 0.05$, however, the (dashed) curves cross the data bands at different values of $\sin^2 \theta_{12}$ (about 0.23 for Ga rate and 0.34 for CC/NC). This mismatch leads to an upper bound on $\sin^2 \theta_{13}$ as in Eq. (5).

In the right plot of Fig. 8 (“renormalized” cross section), this mismatch exists already at $\sin^2 \theta_{13} = 0$, and becomes worse for $\sin^2 \theta_{13} > 0$: there is no value of $\sin^2 \theta_{12}$ which accommodates well both Ga and SNO data with their predictions. In this case, there is a disturbing “tension” between low- and high-energy solar ν data. The tension would be even stronger if the new (lower) solar metallicity ²²⁾ were adopted, since it would further decrease the expected Ga rate (not shown).

Figure 9 shows this tension in an alternative way, by comparing the regions allowed at 1σ by Gallium data (slanted band) and SNO data (closed region) in the usual mass-mixing plane, for four increasing values of $\sin^2 \theta_{13}$. As in Fig. 8, the left (right) panels refer to standard (“renormalized”) Gallium cross section. For standard cross section (left), the Ga and SNO allowed regions fully overlap for $\sin^2 \theta_{13} = 0$, but they increasingly separate for increasing values of $\sin^2 \theta_{13}$. This tension leads to meaningful upper bounds on $\sin^2 \theta_{13}$ [Eq. (5)]. For “renormalized” cross section, however, there is no overlap between Ga and SNO regions at 1σ , even at $\sin^2 \theta_{13} = 0$: there is always “tension”. Therefore, at face value, one would obtain a formally stronger upper bound on $\sin^2 \theta_{13}$ from solar data (not shown), at the price of a worse best-fit at $\sin^2 \theta_{13} = 0$.

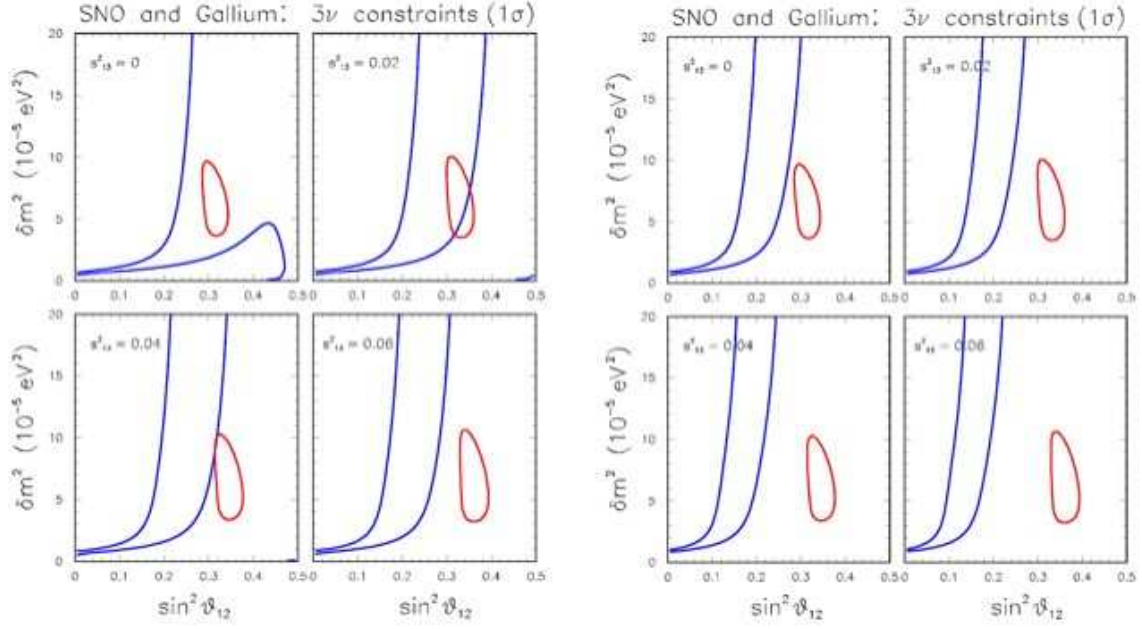


Figure 9: Regions separately allowed at 1σ in the mass-mixing plane by SNO (closed regions) and Gallium data (bands), for $\sin^2 \theta_{13} = 0, 2, 4, 6 \times 10^{-2}$. The four plots on the left (right) refer to standard (“renormalized”) Ga cross section. See the text for details.

In conclusion, if the claim in favor of a lower Ga cross section ³¹⁾ is valid, the current good agreement between low and high energy solar neutrino data would be somewhat spoiled, and the solar ν indication for small θ_{13} would become stronger at face value, but also more ambiguous. Therefore, we think that further work is required to interpret the results in ³¹⁾ before including them in global analyses (through, e.g., a reduced Ga cross section for $E_\nu < 2$ MeV). Certainly, such results provide one more motivation to explore the low-energy spectrum of solar neutrinos and to revisit the ν_e absorption cross section in gallium: two topics (among many others) where John’s contribution will be greatly missed.

4. Conclusions

We have reviewed the status and the successes of solar neutrino research, in the light of the outstanding contributions by John Bahcall. Solar neutrinos provide us with solid evidence in favor of ν masses and mixing, in a beautiful synthesis of physics and astrophysics. But, as for any synthesis, we are eager to go beyond it: more accurate studies might be already revealing something unexpected, e.g., concerning the metallicity problem, or the Gallium cross section. Time will tell us if these or other “disturbances” will disappear as accidental fluctuations, or will instead evolve as new, independent “solar neutrino problems” requiring new (astro)physics.

5. Acknowledgements

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